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PREDICTION OF COINCIDENCE FREQUENCIES FOR A THIN-WALLED, FLUID-FILLED PIPE

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Prediction of Coincidence Frequencies for a Thin Walled, Fluid Filled Pipe

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Abstract

This technical note outlines the role of piping-based noise generation and transmission in naval applications. It describes the nature of turbulence and how it generates acoustic energy. Coincidence between higher order acoustic modes and resonant structural modes is defined and its role in determining the efficiency of generating pipe wall vibrations by internal pressure fluctuations is discussed. The inertia effect of contained liquids is discussed. Coincidence data are presented for a 100 mm NB schedule 40 steel pipe, and the implications for naval applications discussed.

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1. Introduction

The requirement of modern navies for their ships to operate in stealth environments places a demand on ship designers to minimize the noise levels emitted by these vessels. Reduced noise emission assists a vessel to both avoid detection and to enhance "listening" capabilities.

One contribution to a ship's noise "signature" derives from fluid conveyed in pipes and the vibration of the pipes themselves. This noise is often directly transmitted to the external environment through fluid discharge ports or through hull vibration caused by the vibrating pipes. In order to quantify the magnitude and frequency distribution of the externally radiated sound, some understanding of the excitation of pipe wall vibrations by fluid-borne acoustic and turbulent pressure fluctuations is required. Knowledge of the transmission of the induced pipe wall vibrations along the pipework and to surrounding structures is also needed.

The purpose of this report is to elucidate the mechanism of energy transfer from the fluid to the pipework and predict the response of 100 mm nominal bore schedule 40 fluid-filled pipe (114.3 mm OD, 6.02 mm w.t.) to the first three higher order modes of internal acoustic vibration.

The mechanism of the transmission of the structure-borne vibration to the hull wall and the resulting frequencies of the induced hull wall vibrations will be the subject of a subsequent report.

2. Random Pressure Fluctuations in a Fluid

Laminar flow of a fluid in a pipe occurs when the flow path of a given fluid element is predictable. However under certain conditions laminar flow gives way to turbulent flow, composed of fluctuating eddies. The conditions which determine this transition depend on the magnitude of the flow disturbance such

as the pipe surface roughness or other flow restrictions, and the Reynolds number of the flow which takes account of pipe diameter, flow velocity and the viscosity of the fluid. The Reynolds number is a measure of the ratio of the mass inertial forces to the viscous forces in the given fluid flow.

The very nature of turbulent flow causes pressure fluctuations at the internal surface of the pipe causing the pipe wall to vibrate. However only those components of the pressure fluctuations which excite resonant modes of pipe vibration will contribute to the excitation of the pipe wall. For the present application to ship board systems, the contribution to pipe vibration from turbulent pressure fluctuation in a fluid is much less significant than the contribution from acoustic pressure fluctuations within that fluid [1].

3. *Acoustic Pressure Fluctuations*

Any severe disturbance to fully developed turbulent pipe flow (such as a flow restriction, etc.) will result in the generation of a broad band of internal sound or acoustic waves [2]. The dominant source of pipe flow noise and vibration derives from those waves which correspond to higher order acoustic modes. Since it is the resonant flexural modes of the pipe wall which contribute most to the pipe vibration, the overall response of a piping system is predominantly determined by the interrelationship between higher order acoustic modes in the fluid and resonant flexural modes of the pipe wall. This interrelationship consists of two components, the joint acceptance function and the frequency response function (receptance) of the structural mode.

4. *Coincidence Between Higher Order Acoustic Modes and Resonant Structural Vibration Modes*

The joint acceptance function expresses the degree of spatial coupling (wavelength matching) that exists between the (p, q)th acoustic mode in the fluid and the (m, n)th flexural structural mode of the pipe (refer Figs 1 and 2). This matching occurs in both the circumferential and axial directions and complete coincidence is said to occur when there is matching in both wavelength (or wavenumber, which is inversely proportional to the wavelength) and frequency between the modes of the propagating internal sound waves and the resonant flexural modes of the pipe wall.

The receptance is the inverse of the dynamic stiffness of the pipe and so gives an indication of the ease with which a particular frequency can excite the pipe. It has a sharp maximum at the resonant frequency for any given structural mode.

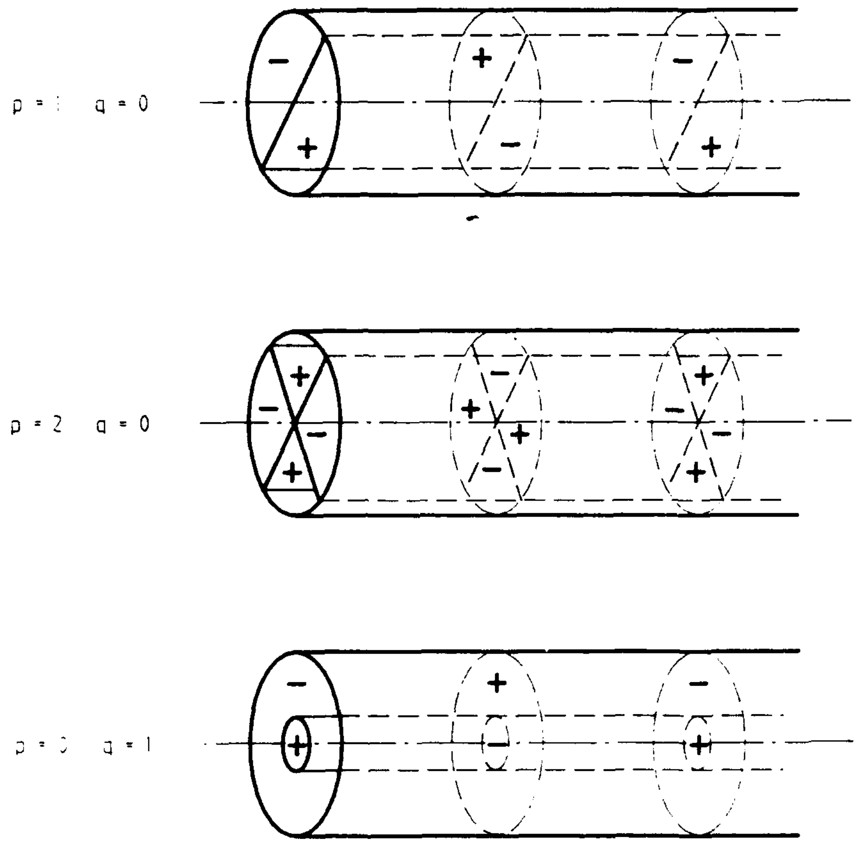

Figure 1: *Acoustic modes within a cylindrical body of fluid.*

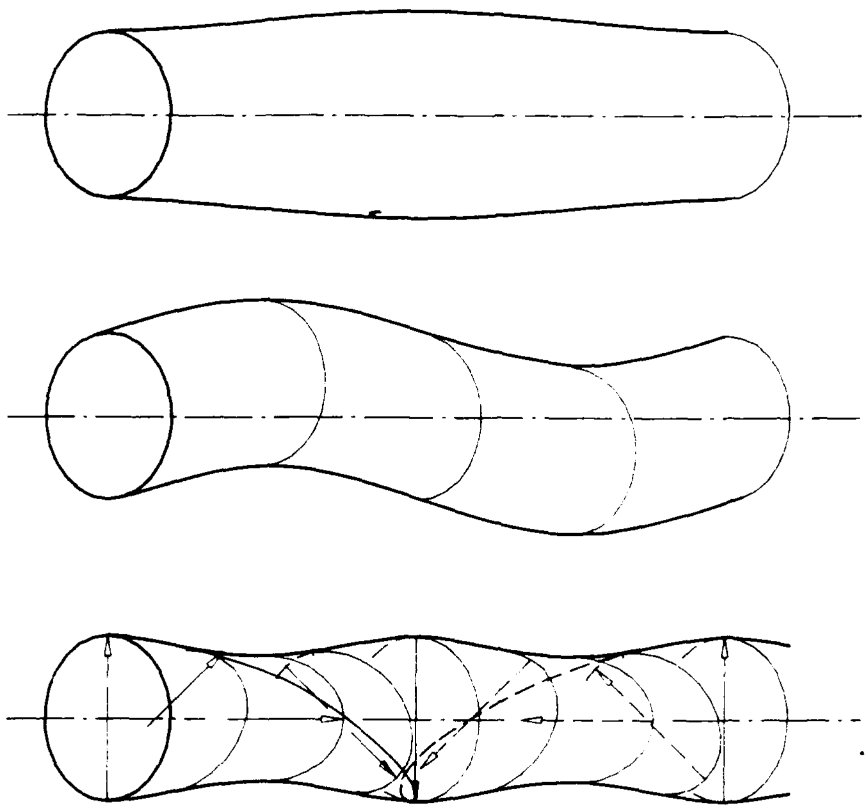

Figure 2: *Flexural modes of vibration for a cylindrical pipe.*

The first three higher order acoustic modes are shown schematically in Figs 1(a) to (c). While the circumferential wavenumber of a propagating sound wave is fixed, it will exhibit a continuous variation in axial wavenumber with frequency. Mathematically these waves can be expressed as follows [3]:

$$P(r,\theta,x) = \sum_{p}\sum_{q} (Apq\cos p\theta + Bpq\sin p\theta)J_p(\phi_{pq},r)e^{i(kx-wt)} \qquad (1)$$

where r, θ and x are the usual cylindrical polar co-ordinates, w is the angular frequency of the longitudinal wave component and Apq and Bpq are constants. J_p is a Bessel function of the first kind of order p, and ϕ_{pq} are the eigen values satisfying the rigid wall boundary condition $J'_p(\phi_{pq}, a_i) = 0$ where J' is the first derivative of the Bessel function with respect to r, and a_i is the internal radius

of the pipe. The first two terms on the right hand side describe the circumferential wave component and the third term, the axial component.

The wave has p plane diametral nodal surfaces and q cylindrical nodal surfaces concentric with the cylinder axis (see Fig. 1). The wave may be thought of as an axially travelling wave with an amplitude which also varies across its cross-section (through the cylinder axis). These higher order acoustic waves may only propagate at frequencies above their cutoff frequencies, i.e.

$$(f_{co})_{pq} = \frac{\Phi_{pq} C_i}{2 \pi a_i} \tag{2}$$

where c_i is the speed of sound in the fluid.

Resonant structural vibration modes also have circumferential and axial wavenumber components although for a particular mode, not only will the circumferential wavenumber be fixed by n, the integral number of full wavelengths around the circumference but, due to the finite length of any typical pipe section (i.e. between pipe supports) the axial wavenumber will assume discrete values corresponding to the number (m) of half waves. The resonant frequency of the (m, n)th structural mode of a pipe wall of thickness t and length L is given by equation (3), [3].

$$f_{mn}^2 = \left(\frac{c_L}{2 \pi a_m}\right)^2 \left(\beta k^4 + (1 - \nu^2) \frac{k_m^4}{k^4}\right) \tag{3}$$

where c_L is the longitudinal wave speed in the pipe wall, ν is the Poissons ratio, $k_m = m\pi a_m/L$ is the axial wavenumber, $k = (k_m^2 + k_n^2)^{½}$ is the effective wavenumber, $k_n = n/a_m$ is the circumferential wavenumber, and $\beta = t/(2\sqrt{3}a_m)$ where a_m is the mean pipe radius.

The first term in the parentheses on the right hand side of equation (3) is associated with the flexural strain energy of the vibrating pipe and the second term is associated with the membrane strain energy. Several different equations have also been developed to predict the frequency of resonant structural modes of a pipe based on different ways in which the strain energy can be defined [4]. Leissa [4] shows these equations to largely agree with the predictions of equation (3).

Equation (3) however predicts the resonance frequencies of a cylindrical shell which contains air at ambient temperature and pressure [3] whilst the present work is concerned with water filled pipes. The substitution of water for air in a vibrating pipe may be interpreted as effectively increasing the mass, or density, of the pipe wall as this will experience a greater inertia to the passage of a flexural vibration. Clinch [5] derives an expression for the effective density of the shell wall due to this "virtual" mass. In the present case this change in effective density is evident in equation (3) only through its effect on the sound velocity of compressional waves in the pipe wall, c_L, i.e.

$$c_L^1 = c_L/F$$

The virtual mass factor, F, is shown by Clinch [5] to reduce from a value of approximately 2 at 400 Hz towards unity at frequencies above 20 kHz. However, due to the approximate correlation between Clinch's values and those for the present case, we adopt a value of $F = 2$ for these calculations in order to predict lower limit frequencies for coincidence.

Whilst in principle complete coincidence requires exact spatial and frequency coupling between the acoustic and flexural modes, in practice this is achieved in the circumferential direction only. Moreover the maximum structural response of coincident modes does not necessarily occur precisely at wavenumber coincidence but at a frequency very close to it, and is critically dependent on both the modal joint acceptance and the modal receptance.

5. *Prediction of Coincidence Frequencies*

A combined plot of axial wavenumber versus frequency for both the higher order acoustic vibrations and the flexural resonances of the pipe wall, equations (1) and (3) respectively, evaluated for a fluid-filled 100 mm nominal bore schedule 40 pipe, enables predicted coincidence points between these two vibration phenomena to be represented graphically. Figure 3 is such a plot where the frequency axis has been non-dimensionalized by the pipe ring frequency, so that curves could be drawn that are independent of the compressional wavespeed in the pipe wall.

Complete coincidence occurs where there is exact matching between the continuous curves of the acoustic modes and the discrete values of the flexural resonance modes. Whilst complete coincidence is not observed in Figure 3, near coincidence occurs at two locations. The frequencies corresponding to these two points are 11.2 kHz and 15.7 kHz for air-filled pipe and, using $F = 2$, 5.6 kHz and 7.9 kHz respectively for water-filled pipe.

If there is uniform fluid flow within the pipe when the frequencies of the acoustic waves are subjected to a Doppler shift. This frequency shift is dependent on the velocity of the fluid flow, U, through the Mach number $Ma = U/c_f$. For shipboard systems $Ma \sim 10/1500 = 0.0067$ and so the Doppler shift of Mk_z ($\sim$ 100 Hz) has little influence on the coincidence frequencies noted above.

6. *Significance for Naval Ship Noise*

The coincidence frequencies identified above represent the frequencies at which vibrations originating in the fluid can be efficiently transmitted along the pipework as structural vibrations. These vibrations may then follow structural paths to the ship's hull where they are radiated to the ocean as sound. However sound which is transmitted to the ocean is attenuated to various extents depending on the frequency of the sound and the distance travelled by the sound [6]. Sea water is an efficient absorber of high frequency sound with

frequencies between 5 and 100 kHz activating an ionic relaxation of the magnesium sulphate contained in salt water. Frequencies of 10 kHz are attenuated at a rate of approximately 1 dB per kilometre compared with 0.01 dB/km for a 500 Hz frequency [6]. Therefore, frequencies of coincidence predicted in the previous section, if transmitted directly to the ocean, would be attenuated at this greater rate.

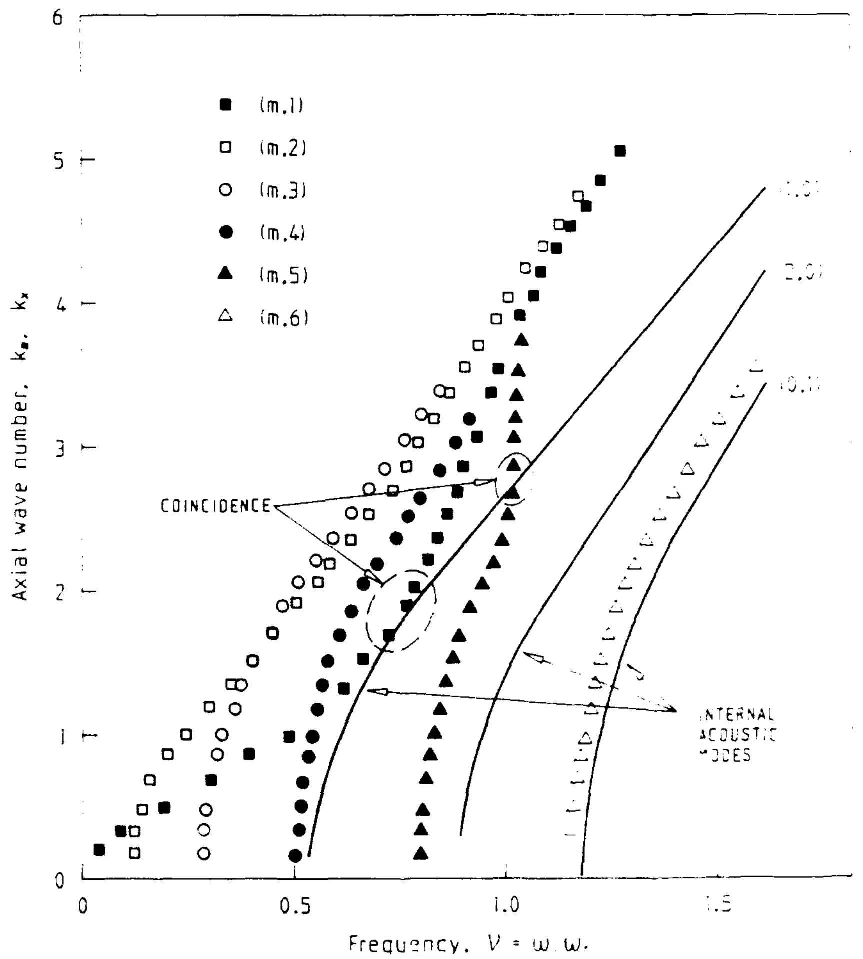

Figure 3: Plot of axial wavenumber versus frequency for both acoustic (k_x) and flexural (x_m) vibrations.

Consequently the contribution to a ship's long range noise signature from coincidence frequencies, similar to or higher than those identified here, will be small.

7. Conclusions

The transmission of fluid-borne noise to 100 mm nominal bore schedule 40 pipework is likely to occur at frequencies at least as great as 5.6 and 7.9 kHz. These frequencies, if transmitted to the ocean however, are attenuated at a much higher rate than are lower frequencies below 1 kHz.

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